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Inverse Square Law for Light and Radiation: A Unifying Educational Approach

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***Abstract*—Many concepts in the physics curricula can be explained by the inverse square law. Point-like sources of gravitational forces, electric fields, light, sound and radiation obey the inverse square law. This geometrical law gives the ability of unifying educational approach of various cognitive subjects in all the educational levels. During the last years we have been using engaging hands-on activities to help our students in order to understand the cohesion in Nature and to export conclusions from experimental data. The development of critical thinking is also stimulated by student 's experimental activities. Teaching students to think critically is perhaps the most important and difficult thing we do as science teachers. In this paper three activities are described, which were executed by students. These activities are concerning the electromagnetic radiation and the main goal is to confirm the inverse square law. We used three activities entitled as: "Inverse Square Law-Light", "Photometer construction" and "Radioactive source". The significant motive for this work constituted the following question: "Is it possible to find lab activities which bring out unification and a non-piecemeal description of physical phenomena, helping students to think critically?".**

***Index Terms*—Inverse Square Law; Light; Photometer; Radiation; Science Teaching.**

I. INTRODUCTION

The radiation intensity from a point-like source with unlimited range, which effects in all directions, in a specific distance r is equal to the quotient of the power to the surface of an imaginary sphere with radiant r.

In the following figure, I is the intensity in r distance, that corresponds to a surface A. At a 2r distance the same amount of energy pass through the surface 4A. So the intensity becomes I/4 etc.

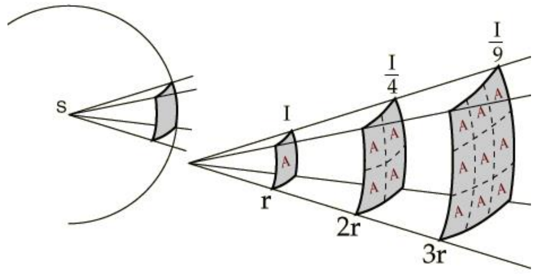

Fig.1. A specified physical quantity or intensity is inversely proportional to the square of the distance from the source of that physical quantity.

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Therefore, the power is proportional to the inverse square of the distance. Being strictly geometric in its origin, the inverse square law applies to diverse phenomena. Newton's law of gravity, Coulomb's law for the forces between electric charges, light, sound and radiation obey the inverse square law. This geometrical law gives the ability of unifying educational approach of various cognitive subjects in all the educational levels. This paper describes simple experiments that verify the inverse square law.

Students know intuitively that intensity decreases with distance. A light source appears dimmer and sound gets fainter as the distance from the source increases. The difficulty is in understanding why the intensity decreases as 1/r2 rather than as 1/r or 1/r3, or even as 1/ √r, where r is the distance from the source.

In a recent paper [1] it is shown how to obtain the inverse-square law of the distance to the light intensity emitted from a small source in a simple, fast and with good precision way. In another recent paper P. Papacosta and N. Linscheid describe a simple experiment that verifies the inverse square law using a laser pointer, a pair of diffraction gratings, and a ruler [2].

The development of critical thinking (CT) is widely claimed as a primary goal of science education [3]. A method for development of critical thinking skills is the Socratic questioning method. Its implementation provides opportunity to help students in appropriate manner to understand concepts and phenomena. The development of critical thinking is also stimulated by student 's experimental activities. For the educational approach of the different actions that take place in this paper, we suggest the educational model that includes the following steps: 1. Trigger of interest 2. Hypothesis expression 3. Experiments – Measurements, 4. Formulation of conclusions and proposals - recording 5. Generalisation - feedback – control.

It is an important part of learning that a person sees and engages a concept several times before mastery is attained [4]. This is very useful in clarifying concepts, as well as when predicting the course of the experiment and its subsequent explanation. An example is the inverse square law.

II. EXPERIMENTS

A. 1st Experiment: Inverse square law – Light

1) Materials

A cardboard with grid, a cardboard with a hole, supporting clips, ruler, candle.

Students set the device shown in the following picture so that the cardboard with the hole to be at the middle of the distance between the candle and the cardboard with the grid.

They observe and they count the lighted squares on the cardboard with the grid.

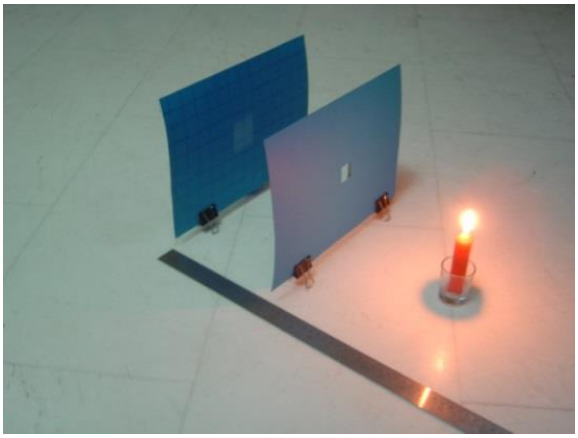

Fig. 2. The apparatus used in the 1st experiment.

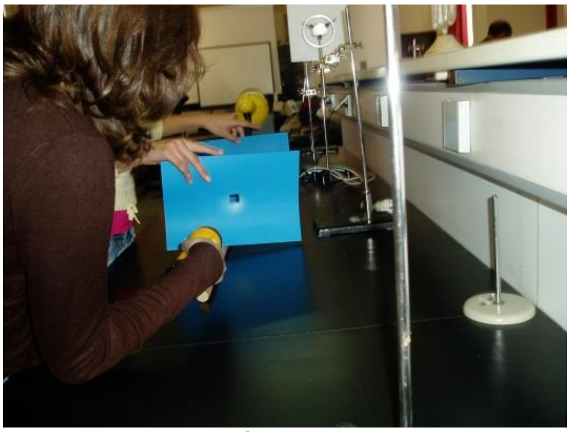

Fig. 3. The 1st experiment.

We can make, for example, the following questions to the students for hypothesis expression from them.

What do you think will happen if we redouble the distance between the first cardboard with the hole and the second one with the grid?

When the distance between the candle and the hole is equal to the distance between the hole and the cardboard with the grid, how many squares are lightened?

2) Procedure

1) Keep the distance between the bulb and the card with the 1 cm square hole constant at 10 cm. Put the bulb at different distances from the graph paper and count how many squares on the graph paper are lit at each distance. Record the number of squares illuminated in the data table. (Comment: Be sure to measure the distance from the bulb, not the card.)
2) Measure the size of the squares in the graph paper to determine the area of each square. If you use the graph paper provided with this activity they should be 1/2 cm on a side, and thus each has an area of 1/4 cm2. Calculate the area illuminated at each distance measured, and record it in your data table.
3) The amount of light received per area is called brightness. The amount of light given off by the bulb and passing through the hole in the card always remains constant. So, what we want to calculate is the brightness relative to some standard brightness (say the brightness of the bulb on the graph paper at 10 cm). We call brightness B, Area A, and the amount of light (also called power or luminosity) L, and we can write the following:

$B = L/A$ for any distance and $B_0 = L/A_0$ for the standard distance (10 cm)

So relative brightness is $B/B_0 = A_0/A$ (L cancels out because it is the same for both)

But, at a distance of 10 cm the area illuminated was 1 cm^2
So, $A_0 = 1$ and we have $B/B_0 = 1/A$

Calculate the relative brightness for each distance, and record it in your data table.

TABLE I: DATA TABLE OF THE 1ST EXPERIMENT

Distance from bulb (cm)	Number of squares illuminated	Area illuminated (cm^2)	Relative brightness (cm^{-2})
10	4	1.00	1
13	6.7	1.68	0.6
15	9.2	2.30	0.43
17	11.5	2.88	0.35
20	16.5	4.13	0.24
23	22.2	5.55	0.18
25	26	6.50	0.15
27	28.5	7.13	0.14
30	36.5	9.13	0.11

Using the data from the above table students can make the graph of relative brightness vs distance (data as points and plotting the theoretically line). As a conclusion we have that the relative brightness should obeys the low $B/B_0 = k/ r2$. (Comment: The constant of proportionality is k = 1/100, because for r = 10 cm, A = 1 cm2)

B. 2nd Experiment: Photometer construction

1) Materials

Two paraffin blocks, ruler, two similar lightings, four lamps and aluminium foil.

Building a photometer. Verification of the inverse square law for the light. The aim is to create a photometer and to verify the relation between the power of light and distance.

2) Procedure

1) Put the aluminium foil between the two pieces of paraffin.
2) Put the two lamp holders in one-meter distance between them.
3) Both lamp holders have lamps of 100W. Close all the other lightings and put the photometer between the two lamp holders so that the two pieces of paraffin have the same luminosity.
4) Fill the data table.
5) Replace one lamb of 100W with another of 75 W and repeat the second and the third steps.
6) Repeat the second and the third steps with other combinations of lamps and we fill the table.
7) Check if the data (measurements) follows the inverse square law.

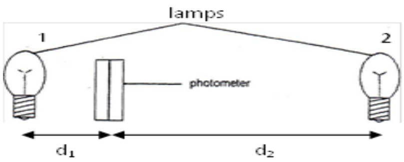

Fig. 4. Description of the 2nd experiment.

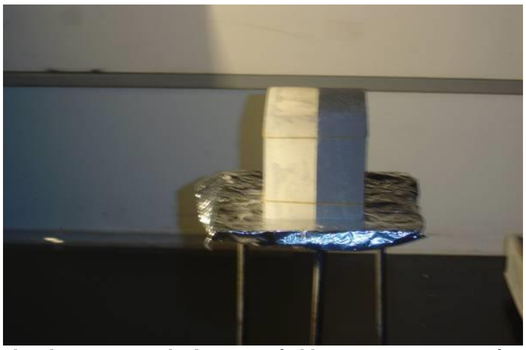

Fig.5. The photometer with aluminum foil between two pieces of paraffin. What the two paraffin pieces will look like if they receive different amounts of light.

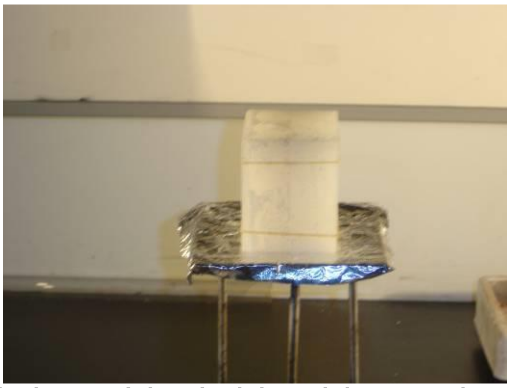

Fig.6. The photometer lightened with the two lightings. How the two blocks of paraffin will appear if they receive equal amounts of light.

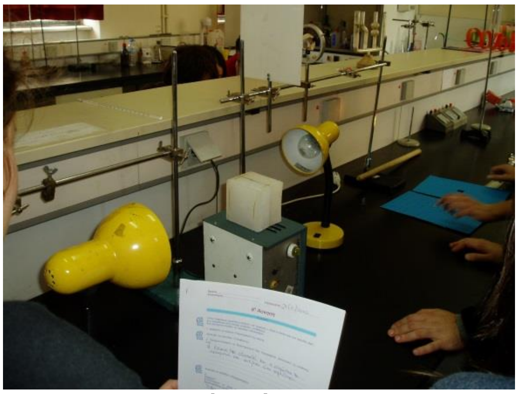

Fig.7. The 2nd experiment.

TABLE II: DATA TABLE OF THE 2ND EXPERIMENT

P_1(W)	P_2(W)	P_1/P_2	d_1(cm)	d_2(cm)	d_1/d_2
100	100	1/1	50	50	1/1
75	100	3/4	46	54	46/54
40	100	2/5	39	61	39/61
40	75	8/15	43	57	43/57

P_1: power of Lamp1
d_1: distance between lamb1 and photometer
d_2: distance between lamb2 and photometer

The same luminosity means the same intensity I of light incident on each one of the paraffin blocks. If the intensity I= k / r² (k a constant depends on source accordingly from its power).

For the second case (P_1=75W, P_2=100W), P_1/P_2=3/4. From the experimental data it emerge that intensities are equal at distances d_1=46cm and d_2=54cm. When the intensity is the same on both paraffin blocks (as shown in Fig. 5) then these two intensities can be put into an equation. So we have:

$I=k/(d_1)^2$ $\quad$ $I=k'/(d_2)^2$ $\quad k'=3/4\ k$
$(d_1/d_2)^2 = (d_1/d_2)^2 = (46/54)^2=0.73 \sim 3/4$
The same is for the other two cases.
$(d_1/d_2)^2 = (39/61)^2=0.41\sim 2/5$
$(d_1/d_2)^2 = (43/57)^2=0.57\sim 8/15$

Thus the law is verified.
(Comment: There is an error of about 7%. The theoretical reading of the ratio of the two intensities (using light bulbs of 40W and 75W) should be 0.53 and not 0.57 as measured. One reason for this is that the students did not take into account the fact that the overall luminous efficiency (% of light energy/heat) of incandescent light bulbs changes with the wattage of the bulb. For example, a 40W tungsten incandescent light bulb has a luminous efficiency of only 1.9% (only 1.9% of its 40W power is converted into visible light). For a 60W light bulb the luminous efficiency is 2.1% and for a 100W light bulb is 2.6%.)

C. 3rd Experiment: Radioactive source.

1) Materials

Radio-active Cobalt-60 5μCi, Geiger-Müller, ruler.

The inverse square law in a radioactive source of gamma rays, using a Geiger- Müller is studied. The aim is to ascertain the validity of the law also in electromagnetic radiation that emits from radioactive sources.

2) Procedure

1) Record the measurements from the Geiger-Müller for two minutes.
2) Repeat the measurement four times and calculate the mean rate per minute.
3) Rotate the tube of the meter 900 (it is to eliminate any effect of Alpha and Beta particles that may distort the reading of the Gamma rays) and repeat 2 and 3 steps.
4) Compare the results from the different directions of the meter. This is the stand radioactivity.
5) Put the Geiger 8 cm away from the source.
6) Measure for every minute.
7) Repeat step 3 for 16 cm, 24 cm and 32 cm.
8) Check if the data (measurements) follows the inverse square law.

Using the Geiger Muller we took stand radioactivity measurements for two minutes in two vertical directions. There has been taken five different measurements in each direction.

Continuously we used a radioactive source Cobalt-60 5μCi and took five measurements for 2 minutes period in two different distances 20 cm and 40 cm. The data confirmed satisfactory the inverse square law.

TABLE III: DATA TABLE OF THE 3RD EXPERIMENT

Intensity I (lux)	Distance r (m)
160	0,42
140	0,50

130	0,52
120	0,56
100	0,62
87	O,67
80	0,70
60	0,84
40	1,12
30	1,40

Measurements are with the radioactive source of Cobalt-60 5μCi without the background radiation.

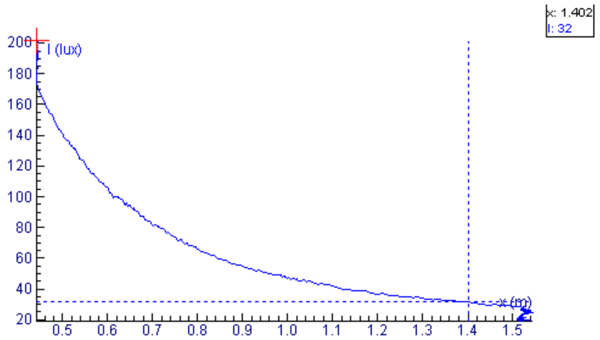

Fig. 8. Graphic plot of intensity (I) vs distance (r).

As a conclusion we have that the intensity of gamma rays radiation decreases as we go away from the source of radiation and obeys the low I=k/r2.

Safety and technical notes: Note that 5μCi is equivalent to 185 kBq. Cobalt-60 is the best pure gamma source. However, students can use sealed radium source. This gives out alpha, beta and gamma radiation. Students can use it for this experiment by putting a thick aluminium shield in front of it. This will cut out the alpha and beta radiations. An alternative is to try using a Geiger-Muller tube sideways. The gamma radiation will pass through the sides of the tube but alpha and beta will not.

III. Generalizations

For generalizations we can apply the following subjects for further study:

1) The inverse square law for gravitational and electrical forces and it's relation to the gravitons and photons respectively.
2) The magnitude of a star. When the Absolute Magnitude of a star is known (as in the case of standard candles) then the distance to such a star can be calculated by the use of the Inverse Square Law. (As Edwin Hubble did in 1924 and 1929. He used the Luminosity – Periodicity law of Cepheid stars discovered by Henrietta Leavitt.). The Inverse Square Law is a powerful tool for astronomers that help to calculate distances to stars and galaxies near and very far away (using Supernovae of the Ia type).
3) The absolute magnitude of a star.
4) The inverse square law for sound. The sound intensity from a point source of sound will obey the inverse square law if there are no reflections or reverberation.

IV. Assessment

By the end of the activity students should be able to [5]:

- Explain what the inverse square low is.
- Identify the mathematical expression of an inverse square low.
- Describe an experiment for checking the inverse square low for the light.
- Do a quick mathematical check for given data (e.g. by doubling and tripling the distance and seeing if the data follows an inverse square law by dropping to a quarter and a ninth).
- Predict a measurement (comparatively) for a given distance from the source.
- Predict the gravitational and electrostatic forces between objects.

The intervention was performed on high school students (17years old) in Athens, Greece during the school year 2016-2017. The number of students participating in this study was forty seven (47) students - two (2) classes, one of twenty four (24) students and the other of twenty three (23) students - divided in sixteen (16) teams of three (3) students each (there was one team of two students). For the assessment of the proposal they took pre, post and final tests. We find that the quality of the students' reasoning about the inverse square low is improved by this approach.

A comment of a student summarizes the main attitude of all students "These activities were particularly interesting and helped us to better understand the concepts learned. All showed interest. I think it is good all students to learn in this way."

V. Conclusion

The activities used to teach students the inverse square low support a unifying approach for this low. The unifying approach enhances learning, helping students to think critically. The development of critical thinking is stimulated by student 's experimental activities which lack strict instructions.

The experiments are carried out by students and can be used for supporting the teaching of the inverse square low in an inquiry- based approach, as well as helping students to approach the nature of science by guiding them to realize the relationship of experiment and theory in scientific investigations and also the way scientists work.

Our didactical approach seems, from the assessment, to be quite encouraging and we suppose that it is appropriate not only for high school students. We think that it will be beneficial and for non-major science university undergraduate students too.

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