European Journal of Engineering and Technology Research
Vol 9 | Issue 3 | June 2024
ISSN 2736-576X

RESEARCH ARTICLE

Liquefaction Analysis of Four High Tailings Dams

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ABSTRACT

This is the story of a long, 3-year study on the investigation of the liquefaction analyses of four tailings dams. All of them were built by the upstream construction method and initially considered unsafe. Before this study, liquid limit stability analyses on these four structures indicated a low factor of safety. This work aimed to carry out stress-strain analyses with the NorSand constitutive model. The work started in 2020 when most stress-strain commercial programs did not yet work with NorSand, but in due course, programs improved and enabled the use of this model. Most of the dams had limited in situ and laboratory tests. In situ tests in most dams were short and did not reach the bottom of the tailings. Existing laboratory tests, on the other hand, aimed only to yield strength parameters, not NorSand ones. The authors then used a mix of strength parameters and literature-recommended typical values. The final analysis results indicated that two of the dams were very unsafe, and two others were very safe. The authors compared the results of the liquefaction potential and safety factors obtained by reducing strength parameters. Despite the theoretical limitations of the parameter reduction technique, a good, inverted correlation was found. The authors concluded that limit equilibrium analyses of tailings may be misleading and should be avoided.

Submitted: May 01, 2024

Published: June 28, 2024

10.24018/ejeng.2024.9.3.3146

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Keywords: NorSand, Stress-strain analysis, Tailings, Tailings dams.

1. Introduction

In the wake of Samarco's Fundão Dam failure (19 casualties, [1]) and Vale's Brumadinho Dam disaster (nearly 300 casualties, [2]), mining companies and the government's dam safety organisations were urged to improve their safety standards and analyses methods.

The standard GISTM [3] brought up a series of best practices subsequently adopted by mining companies. This document recommends using a stress-strain approach to analyse the behaviour and safety of existing tailings dams.

This paper summarizes the experience of the authors and findings during a 3-year analysis work on four iron ore tailings dams, whose names, locations, and owners cannot yet be disclosed. Some of these dams were considered risky based on previous liquid limit analyses, but the stress-strain methods in some cases have shown otherwise.

This text presents the methodology used, encountered difficulties, and some results.

2. Materials and Methods

2.1. Numerical Modelling of Tailings and Computer Programs

Jefferies [4] proposed the NorSand constitutive model, but despite its advantages for modelling sandy tailings behaviour, it took a while before geotechnical software, like Plaxis and GeoStudio, implemented it. By early 2020, only GeoStudio [5] and Flac [6] ran NorSand, while Plaxis [7], [8] brought up a robust NorSand model by late 2022.

After a few failed attempts to implement a NorSand in Plaxis, the Authors started using the GeoStudio suite in the early stages of this project. Later, the Authors shifted to Plaxis 2D software, which has a robust NorSand model and runs very fast.

2.2. Model Parameters

Jefferies and Been [9] present extensive information on how to obtain NorSand parameters (Table I), requiring advanced laboratory testing and the use of a moist tamping technique to prepare the tailings specimens.

TABLE I: NorSand Model Parameters [7]

Parameter	Typical range	Remarks
CSL		
Γ	0.9–1.4	Altitude of CSL, defined at 1 kPa
l	0.01–0.07	Slope of CSL, defined on the natural logarithm
Plasticity		
M_{tc}	1.2–1.5	Critical friction ratio, triaxial compression as the reference condition
N	0.2–0.5	Volumetric coupling coefficient for inelastic stored energy
H	25–500	Plastic hardening modulus for loading, often $f(y)$; as a first estimate for refinement, use $H = //$
χ_{tc}	2–5	Relates maximum dilatancy to y. Triaxial compression as the reference condition
Elasticity		
I_r	100–600	Dimensionless shear rigidity (G_{max}/p')
n	0.1–0.3	Poisson ratio

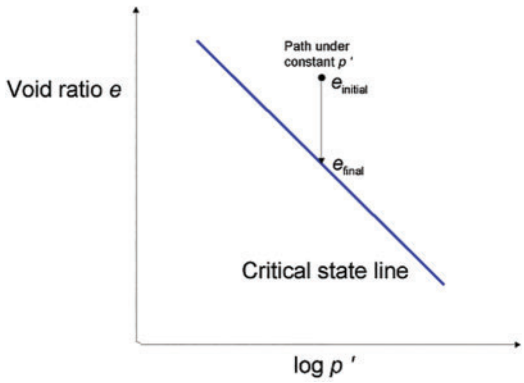

Fig. 1. Definition of the state parameter (ψ), [9].

In addition to the model parameters, one needs the state of the tailings given by the state parameter (ψ) (Fig. 1).

2.3. State Parameter and CPTU

The in situ piezocone test has been the primary tool for obtaining the initial state parameter for loose sandy tailings. Among the existing methods, the Authors investigated references [10]–[12] for comparison. The first one requires a K_0 value and porepressures (u_2) measured by the piezocone, which may be unreliable in interlayered deposits. On the other hand, the Schnaid and You [12] method applies to clean sands and requires a value of shear wave velocity (vs.) or shear modulus (G_0), usually available at 1-metre intervals.

Experience shows that Robertson's [11] method is easy to apply and yields similar results to the Plewes *et al*. [10] method, therefore, being selected for extracting the initial field value of the state parameter.

2.4. Characteristic Value for the State Parameter

One of the challenges of this project was to select a characteristic value for the tailings state parameter. Fig. 2 presents the box plot for the distribution of all CPTUs

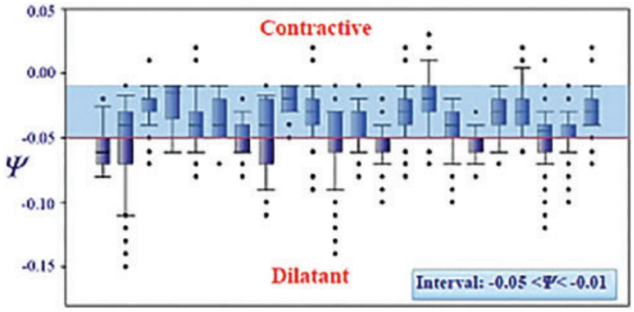

Fig. 2. Box plots of y distribution for all CPTUs.

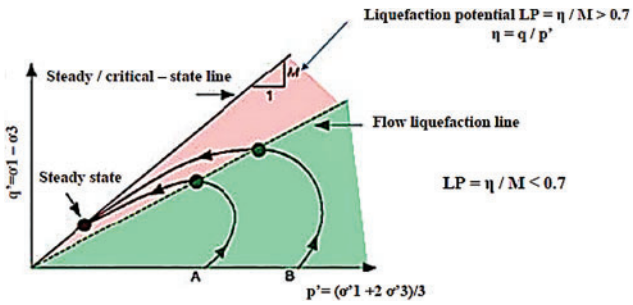

Fig. 3. Definition of liquefaction potential (LP) on the stress path.

for Dam No. 1. It shows fairly homogeneous tailings. The authors then decided to select a characteristic value equal to the third quartile, i.e., 75% of the tailings are more dilative than the selected value.

2.5. Dam Construction Modelling and Results

All four dams were built long ago and were not operating. Construction records were not available. Therefore, the Authors carried out construction modelling using a simple elastoplastic constitutive model with Mohr-Coulomb failure criteria and the MC model [7] with up to 15 construction steps. The measured water level (WL) was then applied to the model, and all displacements were set to nil.

The next stage was to change the constitutive model to NorSand and analyse all results in terms of deformations, liquefaction potential, and SRM (strength reduction method).

2.6. Liquefaction Potential

The liquefaction potential (LP) is a parameter that indicates the proximity of the CSL (critical state line), also called the flow liquefaction line (Fig. 3). LP is given by the ratio:

$$LP = \frac{\eta}{M} \tag{1}$$

where

$$\eta = \frac{q}{p'} \tag{2}$$

LP values close to or above 0.7 indicate a high liquefaction potential and the possibility of the stress path turning to the left due to porepressure rise and drastically reducing strength.

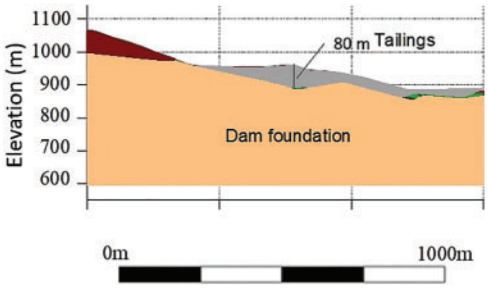

Fig. 4. Cross-section Dam No 1.

2.7. Strength Reduction Method

The strength reduction method (SRM) [13], [14] is a well-known technique in which strength parameters are step by step reduced until large displacements take place in the model. The reduction factor is taken as the strength reduction factor (SRF) value. It works well on simple constitutive models like MC and HSM [7] but has not been implemented in Plaxis for the NorSand model because a more complex model like NorSand may be influenced by other parameters.

Safety analysis using Plaxis uses a so-called phi/c reduction method in which the strength of the soil materials will be reduced with a factor ΣM_{sf} until failure is reached for a stable value of ΣM_{sf}, or the maximum number of calculation steps is reached. When doing a safety analysis using the phi/c reduction method, tan (phi) and c are reduced according to the rule:

$$SRF = \Sigma Msf = \frac{phi_{input}}{phi_{reduced}} = \frac{c_{input}}{c_{reduced}} \tag{3}$$

where phi_{input} and c_{input} are the input parameters, and $phi_{reduced}$ and $c_{reduced}$ are the reduced parameters. For these SRM analyses, the authors also tested the GeoStudio program, which enables SRM calculation.

3. DAM No. 1

Tailings Dam No. 1 (Fig. 4) was raised in 2001 by the upstream method and has not received additional tailings for some 10 years. The dam-raising dykes were built with local residual and compacted soils. The dam is 80 m in height and some 300 m wide. The tailings are loose silty sands overlying saprolites and soft rocks. The dam foundation issues will not be dealt with in this paper.

The site investigation consisted of in situ tests (piezocone CPTU, seismic piezocone CPTUS, vane shear tests, VST) and laboratory tests. From the start of this job, the Authors complained about short CPTUs not reaching the bottom of the tailings (Fig. 5). Nevertheless, the owner did not carry out deep ones.

The laboratory tests initially available enabled the Authors to obtain the strength parameters but were far from yielding all parameters for the NorSand model.

Previous work on this earth structure (Fig. 6) and analysis using limit equilibrium methods (LEM) yielded factors

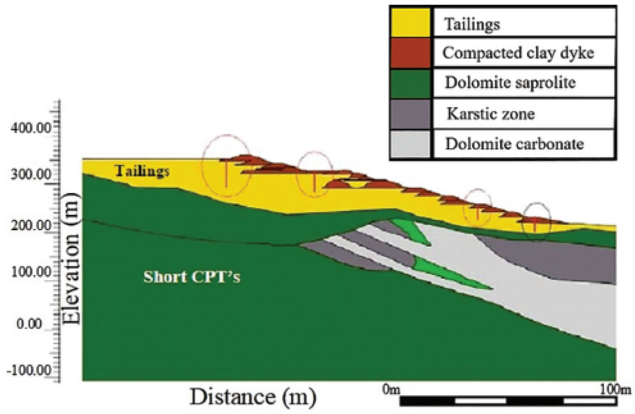

Fig. 5. Short CPTUs along the main cross-section.

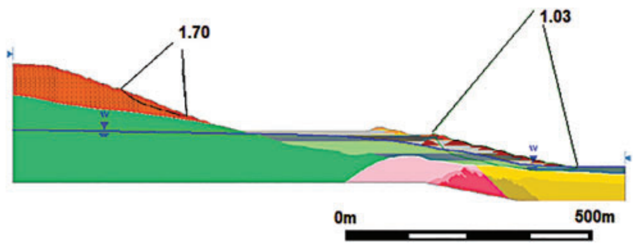

Fig. 6. LEM results from previous analyses (distances in meters).

of safety (FS) as low as 1.03. These analyses were conducted with an undrained strength ratio $\left(\frac{c_u}{\sigma'_{v0}}\right)$ obtained by correlations with CPT data [15]–[17].

The authors then carried out a few drained and undrained laboratory tests on selected samples to extract the NorSand Parameters. Table II shows the resulting parameters. Dilatancy parameters were obtained from the literature-recommended data [9] and the plastic hardening modulus H from curve fitting.

The dam construction was simulated with 15 steps. Then, the water level (WL) was raised to the measured position, and finally, the Mohr-Coulomb model was replaced by NorSand. Fig. 7 shows the result of a Plaxis analysis. The resulting liquefaction potential (LP) values, in most areas, range from 0.3 to 0.4. Some small areas underneath the compacted soil dykes show higher LP values, but the Authors believe that this is due to numerical issues rather than actual high LPs. As previously commented, in addition to performing stress x strain behaviour analyses, the Plaxis program is also capable of carrying out stability analyses (SRM method). However, the program presents as a limitation the application of this method to materials represented by the NorSand model. It was then decided to determine the SRF values by assigning the Mohr-Coulomb model to the tailings.

The result for the final stage of construction presented an SRF of 1.66 (Fig. 8).

To determine a safety factor, a second stability analysis was conducted in which the dyke's strength parameters were not reduced. The program allows this reduction to be avoided in some materials to yield global failure surfaces. This resulted in SRF values of 2.20 (Fig. 9).

TABLE II: NorSand Parameters: Dam No. 1

γ (kN/m^3)	G_{ref} (kPa)	p'_{ref} (kPa)	N	χ	I	Γ	H	M	ψ
24	8000	100	0.3 (*)	3.5 (*)	0.04	1.07	50 (**)	1.22	−0,01

Note: γ: unit weight, G_{ref}: Shear modulus, p'_{ref}: reference pressure, N: Volumetric coupling coefficient for inelastic stored energy, χ: Dilatancy constant, I: Dimensionless shear stiffness (G_{max}/p'): Γ: Altitude of CSL, λ: Slope of CSL, defined on natural logarithm, H: Plastic hardening modulus for loading, ψ: State parameter. (*) typical values for dilatancy, (**) curve fitting.

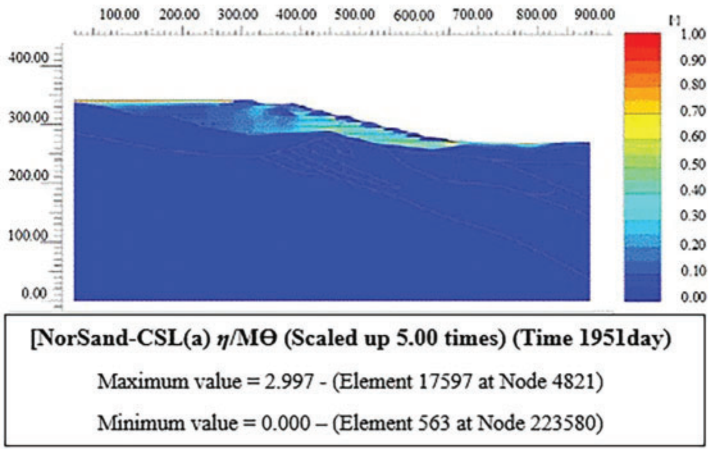

Fig. 7. Liquefaction potential (LP) at the end of construction, Plaxis model.

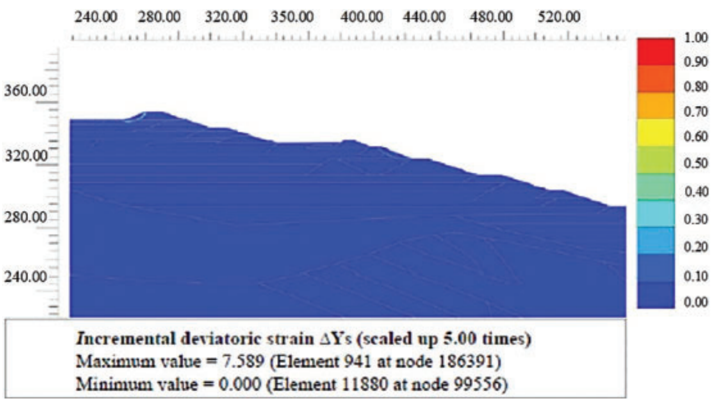

Fig. 8. SRF at the end of construction, Plaxis model.

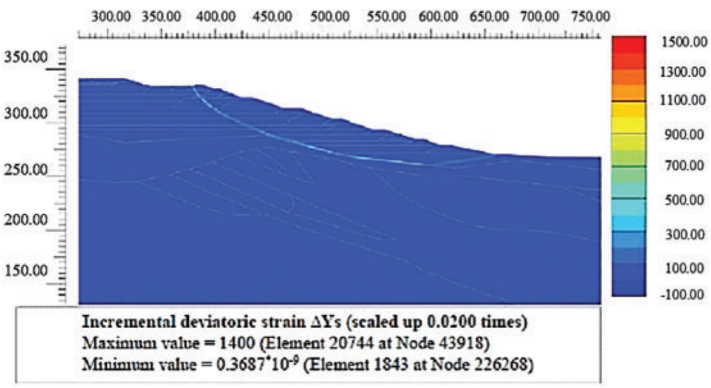

Fig. 9. SRF at the end of construction, Plaxis model–Global.

A parallel analysis of this dam using the GeoStudio suite yielded similar LP values (Fig. 10) and an SRF of 2.2 (Fig. 11).

During this numerical study, the dam owner carried out additional site investigation and detected karst features at the dam foundation. A discussion of these additional data and consequences is out of the scope of this paper, but a collapse possibility was then identified as well as the need for additional investigation and analyses.

4. DAM No. 2

Dam No. 2 is a 55 m high, 150 m wide structure (Fig. 12) which has been closed since 2015. The upstream

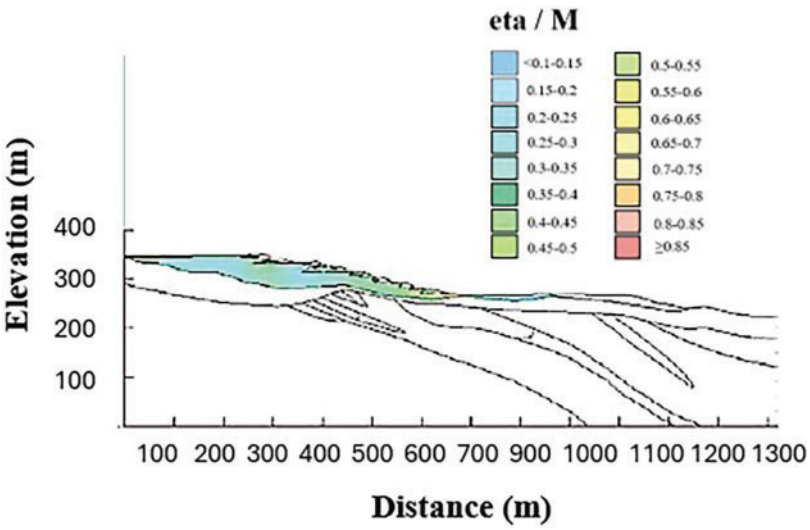

Fig. 10. LP values from the GeoStudio program suite.

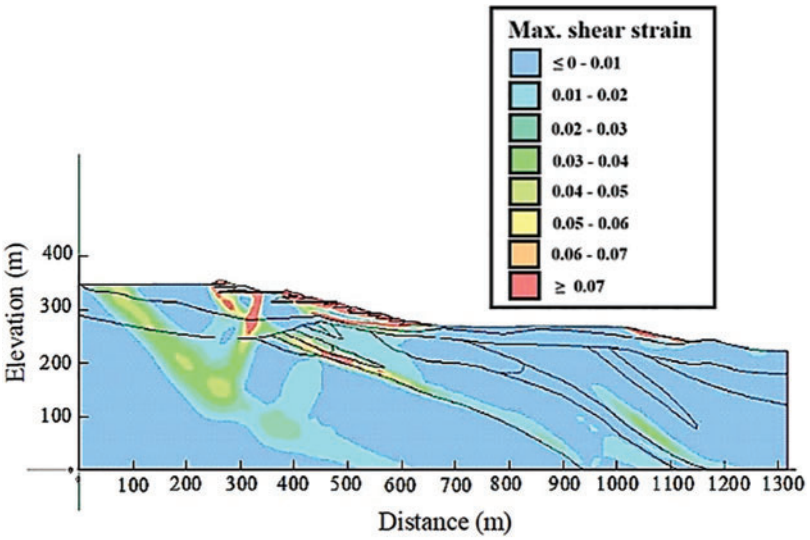

Fig. 11. SRF from GeoStudio program suite.

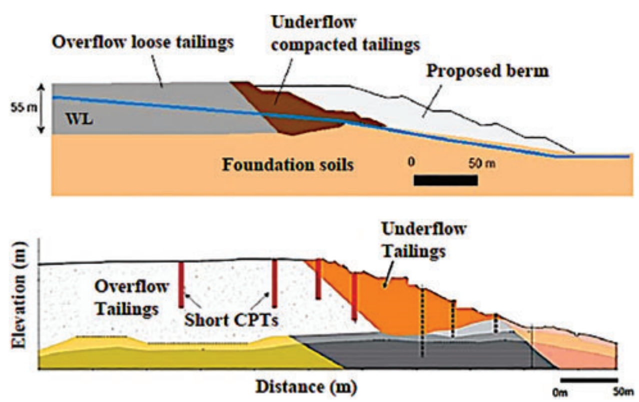

Fig. 12. Cross-section Dam No. 2.

construction method employed sandy iron ore tailings. The buttress or dyke was built with underflow sandy compacted tailings, while the loose overflow tailings were deposited behind.

The existing site investigation data consisted of a series of short CPTs, not reaching the bottom of the tailings and laboratory tests, including characterization and triaxial tests. The foundation soils consisted of competent and high-strength saprolites not described in this paper.

Previous LEM analyses of the above cross-section yielded FSs as low as 1.16 for undrained tailings conditions based on correlations with CPT data.

Fig. 13 shows the initial values and the characteristic value of the state parameter obtained for the loose and compacted tailings.

Like Dam No. 1, the triaxial tests were designed to lead to strength parameters, not fulfilling this study's requirements to yield NorSand parameters. Table III summarises the NorSand parameters used in these analyses.

The dilatancy parameter values in Table III were obtained from bibliography data [9] rather than actual testing.

Fig. 14 presents the model cross-section and the WL-measured position. It also includes the geometry of a stabilising berm, initially proposed by the owner. The work included coupled analyses using FE analysis of water flow through the dam, followed by stress-strain analyses and FS calculation through the methodology described by [18].

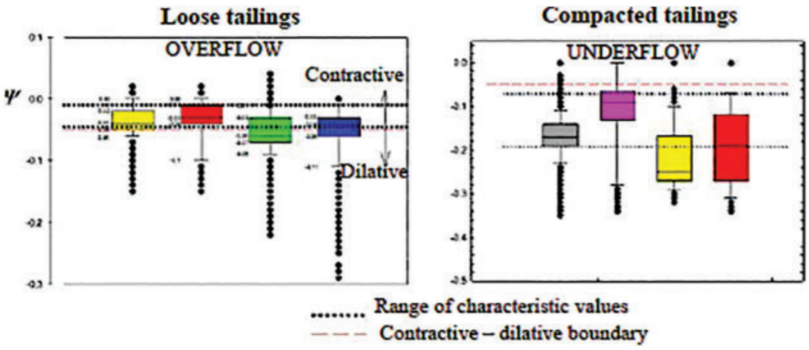

Fig. 13. Characteristic value for the state parameter from CPT data, Dam No. 2.

TABLE III: NorSand Parameters for Loose (Overflow) and Compacted (Overflow) Tailings of Dam No. 2

Tailings	γ (kN/m^3)	G_{ref} (kPa)	p'_{ref} (kPa)	n	ν	N	χ	λ	Γ	H	M	ψ
Overflow	21	8000	100	0, 5	0.27	0.05	3.5	0.03	1	82	1.3	−0.045
Underflow					0.30					80		−0.07

Note: γ: unit weight, G_{ref}: shear modulus, p'_{ref}: reference pressure, n: exponent, ν: Poisson ration, N: volumetric coupling coefficient, χ: dilatancy constant, Γ: altitude of CSL, λ: Slope of CSL, M: critical state parameter, H: Plastic hardening modulus, ψ: state parameter.

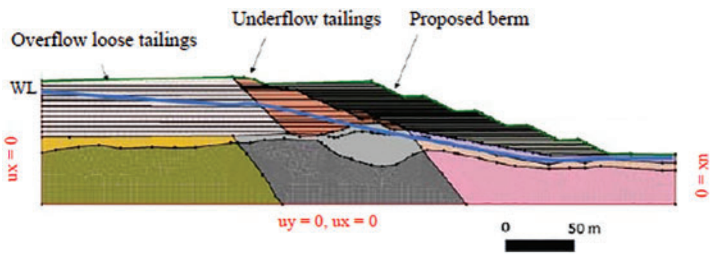

Fig. 14. FEM cross-section and WL.

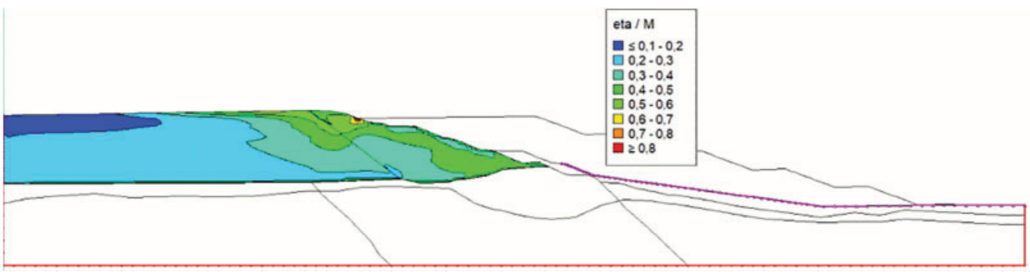

Fig. 15. LP values.

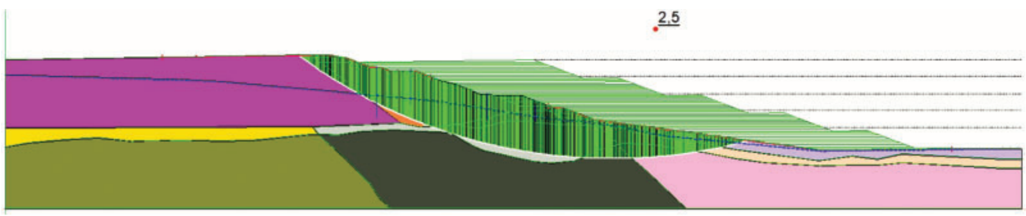

Fig. 16. FS calculation using FE stress, without the berm, resulting in FS = 2.5.

The dam was built in 20 steps with the MC model. Tn WL was raised to its final measured position. At the final stage, the NorSand model replaced the MC model.

Fig. 15 presents resulting LP values showing low values ranging from 0.4 to 0.5. Fig. 16 presents the calculated FS using stresses from the FEM analysis, showing a high FS = 2.5. The analyses clearly show that the owner's proposed berm is not necessary, given that the FS final value is too high.

5. Dam No. 3

Like Dam No. 2, Dam No. 3 (Fig. 17) also has a start dyke of compacted clay and was raised with a buttress compacted underflow tailings. It is a huge structure: 100 m high and some 300 m wide.

Previous LEM analyses lead to very low FS values ranging from 1.28 to 0.84. Given these very low values, the owner proposed a berm reinforcement (Fig. 17) to increase the FS value.

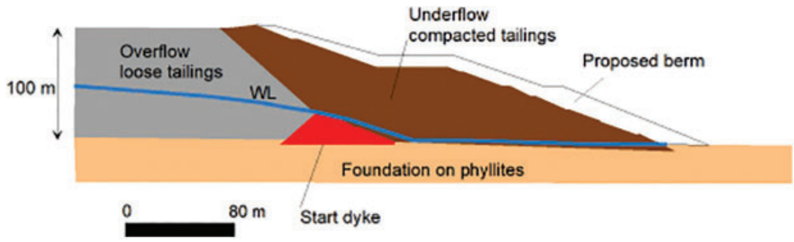

Fig. 17. Cross-section and proposed berm, Dam No. 3.

TABLE IV: NorSand Parameters, Dam No. 3

Tailings	γ (kN/m^3)	G_{ref} (kPa)	p'_{ref} (kPa)	n	ν	N	χ	λ	Γ	H	M	ψ
Overflow	21, 5	8000	100	0, 5	0, 27	0, 05	3, 5	0, 03	1, 0	82, 5	1, 3	−0, 045
Underflow					0, 30					80, 0		−0, 07

Note: γ: unit weight, G_{ref}: shear modulus, p'_{ref}: reference pressure, n: exponent, ν: Poisson ration, N: volumetric coupling coefficient, χ: dilatancy constant, Γ: altitude of CSL, λ: Slope of CSL, M: critical state parameter, H: Plastic hardening modulus, ψ: state parameter.

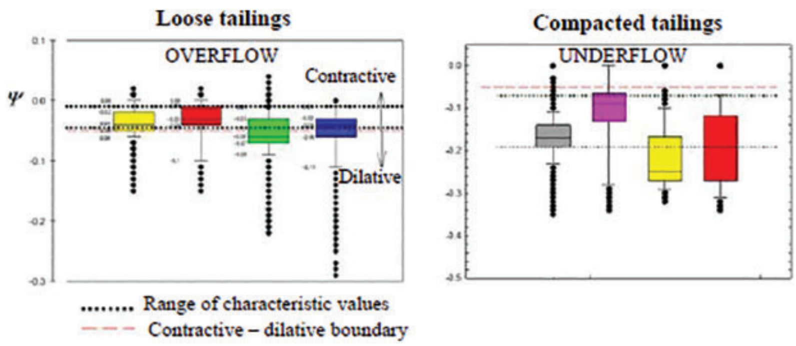

Fig. 18. State parameter from CPTU tests, Dam No. 3.

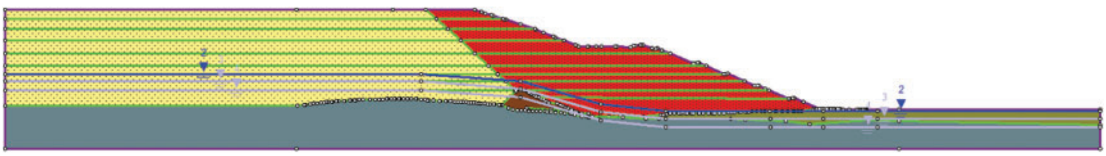

Fig. 19. Model cross-section.

The site investigation included the CPTU tests and a range of laboratory tests, although none were designed to yield the critical state parameters needed for a NorSand analysis. The tailings are predominantly silty sands.

Table IV presents the result of the Author's analysis and the selection of NorSand parameters, in which the dilatancy parameters, likewise the other dams, were based on literature-recommended data.

Fig. 18 shows the range of the state parameter deduced from CPTU tests. For the loose tailings, psi ranges from −0.045 to −0,01, and for the compacted tailings, it is from −0,07 to −0,19. Table IV shows the adopted characteristic values.

The authors used, in this case, the RS2 software, certainly one of the earliest to include the NorSand model. Fig. 19 presents the dam cross-section and the measured WL used for these analyses. The dam construction was simulated in 10 steps using the MC model, eventually replaced by NorSand. RS2 enables SRM calculation only for the MC model. The Authors applied a brute-force method to do SRM with NorSand, reducing the critical state parameter M in 0.1 steps until the program stopped converging.

Fig. 20 shows the SRF automatically calculated by the program with the MC model, resulting in SRF = 1.95, while a similar value was yielded by the NorSand model (Fig. 21) with manual reduction of strength parameter in small steps.

Fig. 22 shows the LP values across the cross-section ranging from 0.3–0.4.

6. DAM No. 4

Dam No. 4 is 55 m high and 210 m wide. This structure was built before 2001. Construction records were lost. After 2015, a waste pile was deposited on the tailings at the back of the reservoir (Figs. 23 and 24).

Table V summarises previous LEM stability analysis results. These calculations yielded a value of FS = 1.3 for a global stability analysis for undrained analyses.

Table VI summarises selected NorSand parameters for the tailings in Dam No. 4. Parameters were extracted from laboratory test results and curve fitting analyses. The

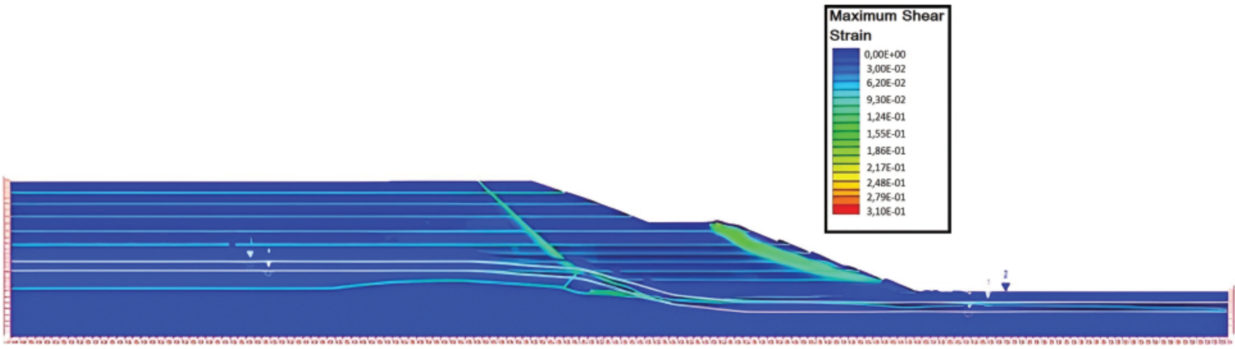

Fig. 20. Max shear strains, corresponding to an SRF = 1.95, MC model.

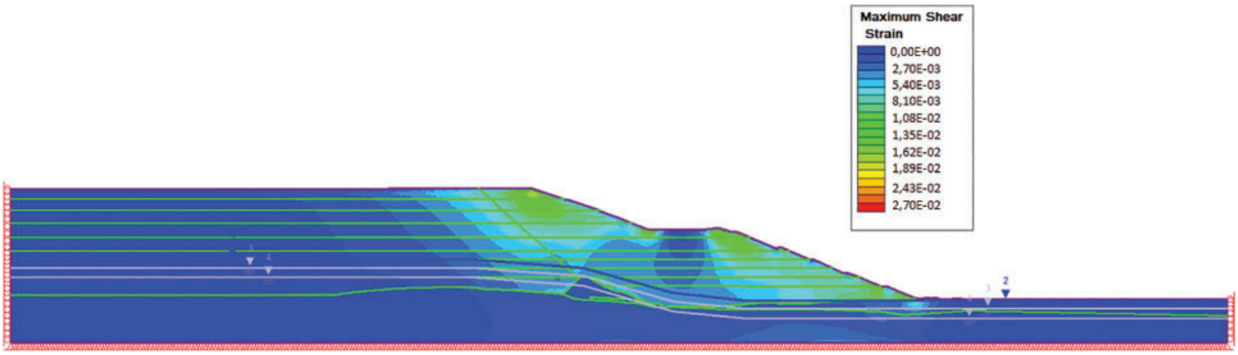

Fig. 21. Max shear strains, corresponding to an SRF = 1.95, NorSand model.

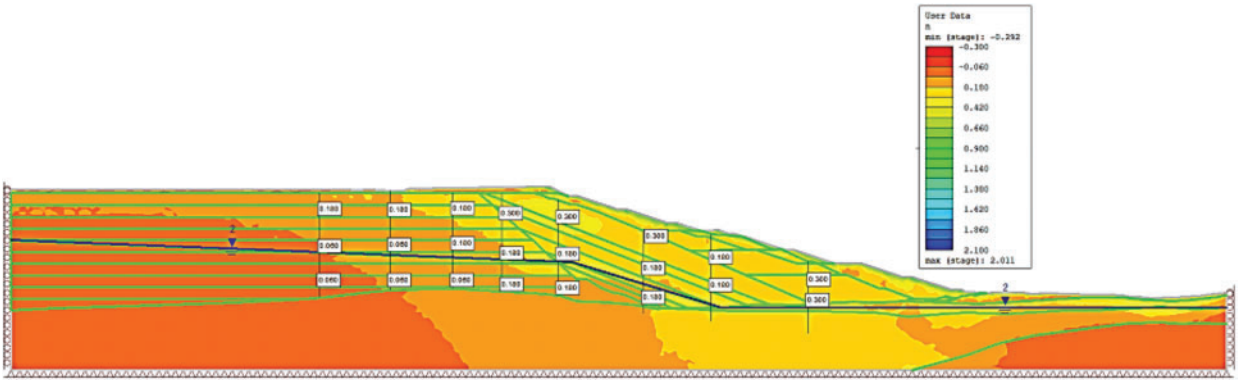

Fig. 22. LP max value 0.3.

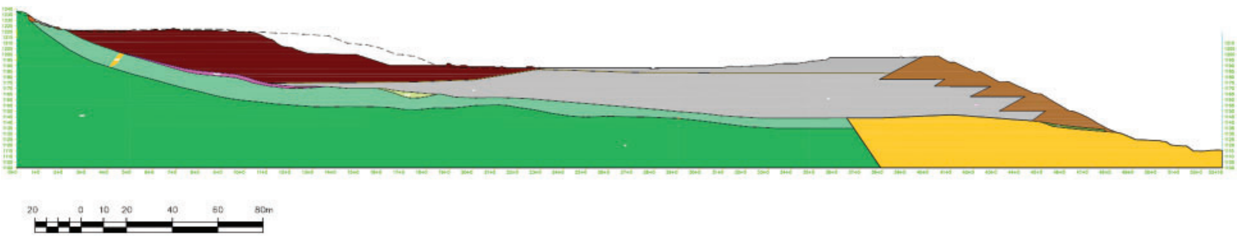

Fig. 23. Cross-section, Dam No. 4.

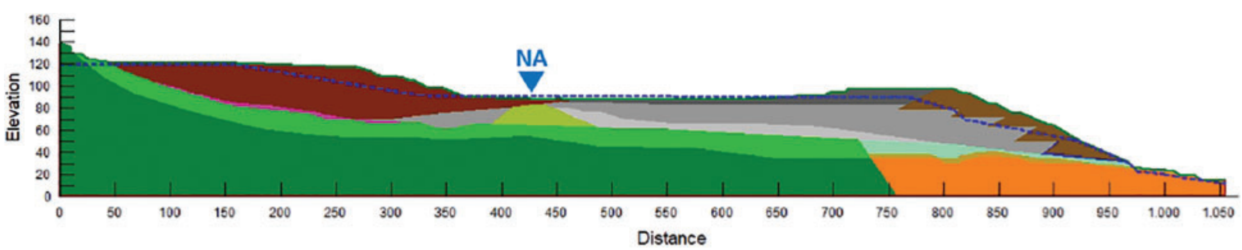

Fig. 24. Measured water level, Dam No. 4.

TABLE V: Summary of Previous LEM Stability Analysis Results

Drainage condition	Undrained strength ratio $\left(\frac{c_u}{\sigma'_{v0}}\right)$	Calculated FS	FS min
Drained	NA	1.28	1.5
Undrained peak	0.36	0.91	1.3
Undrained, residual	0.23	0.40	1.1

Note: σ'_{v0}: Effective in situ vertical stress, c_u: Undrained strength.

TABLE VI: NorSand Parameter Values Adopted for Dam No. 4 FEM Analysis

γ (kN/m^3)	G_{ref} (kPa)	p'_{ref} (kPa)	N	χ	λ	Γ	H	M	ψ
24	9000	100	0.03	3.5	0.01	1.04	1.04	1.34	−0.02

Note: γ: unit weight, G_{ref}: shear modulus, p'_{ref}: reference pressure, n: exponent, ν: Poisson ration, N: volumetric coupling coefficient, χ: dilatancy constant, Γ: altitude of CSL, λ: Slope of CSL, M: critical state parameter, H: Plastic hardening modulus, ψ: state parameter.

TABLE VII: Summary of Main Results

Dam	Height (m)	FS from LE's previous undrained analyses	Liquefaction potential	SRF	Software used
1	80	1.03	0.3 to 0.4	>2	Plaxis/Geostudio
2	55	1.16	0.4 to 0.5	>2.5	RS2
3	100	0.84 to 1.28	0.3 to 0.4	1.95	RS2
4	55	0.91	0.4 to 0.7	1.28	Geostudio

dilatancy parameters N and χ were obtained from typical bibliography values. Apart from the tailings, the other materials like fill, compacted clay, foundation soils, and waste piles were represented by the simple MC model.

Fig. 25 shows a statistical analysis of the state parameter from four CPTUs throughout the tailings. The authors selected a characteristic value corresponding to which 75% of values were higher.

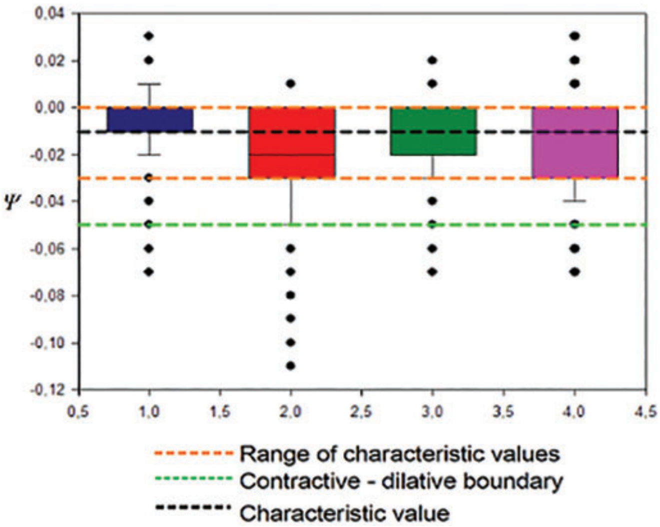

Fig. 25. Selection of a characteristic value for the tailings state parameter from CPTU data.

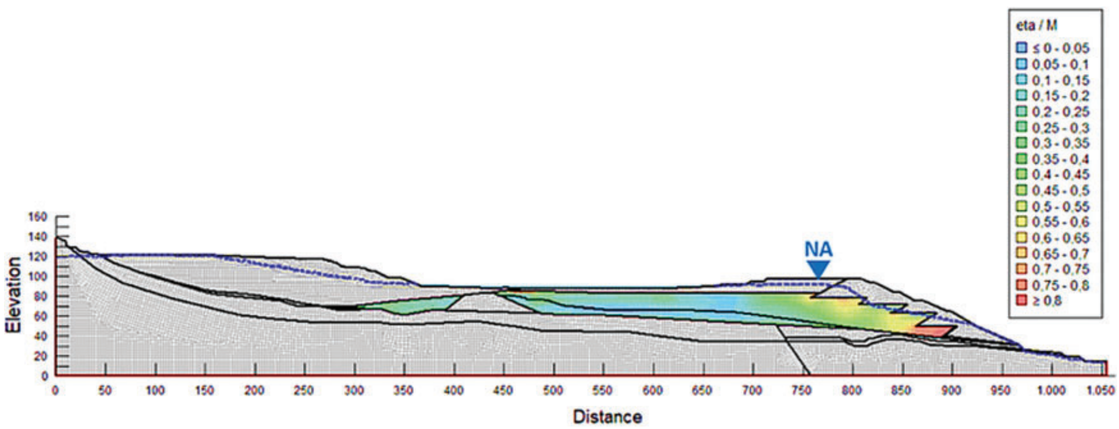

Fig. 26. LP results, couple FEM analysis.

The FEM coupled analysis, i.e., combined flow and stress-strain analysis, was carried out with the GeoStudio program suite using measured water level and permeability values for all materials.

Fig. 26 shows the resulting LP values, which in most dam sections are around 0.4 to 0.5. However, underneath the initial dykes, the LP value is higher and around 0.7, which may be an indication of possible liquefaction.

The associate SRM value calculated by the program is SRF = 1.28.

7. Summary of the Results

Table VII summarizes the main findings of this study of four large tailings dams.

During this study, the Authors faced initial difficulties with some programs which did not run NorSand. However, during this study, program developers improved their software. The main improvement was observed on Plaxis during this study, which became very robust.

8. Conclusions

This stress-strain study of four tailings dams using NorSand model lasted nearly three years and unfolded the following main conclusions outlined below:

1. All dams were built by the upstream method and were considered unsafe. This study showed that two of them (no. 1 and 4) were unsafe. The first one, despite of high safety factor in the tailings, presented karst foundation concerns.
2. Opposite to this, dams No. 3 and 4 are very safe and do not need the reinforcing berms originally proposed by the owner.
3. Limit equilibrium analyses previous to these studies used undrained strength for the tailings. These values were obtained by existing correlations [15]. These analyses usually led to very low FS values.
4. These undrained strength correlations [15] were developed through a limited amount of data from actual failures and may not be applicable to these large dams. Also, LE analysis adopts a constant strength value along the slip surface, which may be a strong limitation for soft tailings. Therefore, LE analysis of soft tailings may be misleading, and the Authors do not recommend their use.
5. During this study, the Authors faced initial difficulties with some programs which did not run NorSand. However, during this study, program developers improved their software.
6. There are theoretical issues for the application of SRM to obtain an SRF value with NorSand. SRM only reduces the strength parameters, while the others remain constant. These unreduced parameters may play an important role, and their sensitiveness should be investigated. Nonetheless, the preliminary assessment of SRM in this study indicates a good inverse correlation between SRF and LP values.

Conflict of Interest

The authors declare that they do not have any conflict of interest.

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